can be observed, the general effect of reduced frequency appears to be described adequately by the assumption made, with the possible exception of data obtained at the lowest Mach number (1.22). The four pitching moment derivatives, as determined by the present analysis, are shown as functions of Mach number in Fig. 2. All experimental data are taken from Ref. 1, where the questions of a similar representation of frequency effects on lift force derivatives and of the resulting axis transfer equations also are discussed.

Concluding Remarks

It is suggested that in cases when frequency effects are quadratic it may be convenient to describe them analytically by introducing second- and third-order derivatives. Such representation, of course, also is more easily adaptable for subsequent application in stability equations. The additional derivatives required can be determined from results of standard oscillatory experiments performed at two frequencies. In addition, at least one more frequency may be required for ascertaining the quadratic form of frequency effects, but these additional experiments can probably often be limited to one representative model at one or two sets of experimental conditions.

Frequency effects, of course, are of interest whenever information is sought which pertains to full-scale conditions featuring relatively high reduced frequencies. It is not equally obvious that frequency effects may also be important in the opposite case, namely when full-scale information involving low reduced frequencies is desired. This occurs when experimental results, because of facilities or techniques available, can only be obtained at high reduced frequency, as is the case, e.g., in short run-duration facilities such as hypersonic shock tunnels or hotshot tunnels. The present method of data analysis and presentation may be very convenient for extrapolating such results to lower frequencies.

Reference

[1] Orlik-Rückemann, K. J., LaBerge, J. G., and Conlin, L. T., "Static and dynamic longitudinal stability characteristics of a series of delta and sweptback wings at supersonic speeds," NRC LR-396, National Research Council, Ottawa, Canada (1964).

Similarity in Axisymmetric Viscous Free Mixing with Streamwise Pressure Gradient

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A SURVEY of the literature on viscous free-mixing with streamwise pressure gradients has been given in Refs. 1 and 2. In the Introduction of Ref. 1, which emphasizes linearized flow, Steiger and Bloom have stated a nonlinear similarity equation for axisymmetric free-mixing with pressure gradients, admitting large velocity defects. They also point out that these types of axisymmetric solution have not yet been treated in the literature. This note presents a brief derivation of the axisymmetric similarity equation in more general form, and gives a particular solution (other than that of uniform flow) which may be expressed in closed form.

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The following boundary-layer equations are assumed to govern the incompressible, axisymmetric viscous free-mixing:

Continuity

$$(ur)_x + (vr)_r = 0 \tag{1}$$

Momentum

$$uu_x + vu_r = (\nu/r)(ru_r)_r + u_e u_{ex} \tag{2}$$

where x and r are, respectively, the streamwise and radial coordinates with corresponding velocity components u and v, ν is the kinematic viscosity, subscripts x and r denote partial differentiation with respect to the indicated variable, and subscript e denotes inviscid conditions at the outer edge of the viscous layer.

The boundary conditions are, at

$$r = 0 \qquad u_r = 0 \qquad v = 0 \tag{3a}$$

$$u = u_e \quad \text{as} \quad r \to \infty \tag{3b}$$

Let

$$\eta = r/N(s) \tag{4a}$$

$$s = x \tag{4b}$$

and define a similarity parameter as follows:

$$u = u_e(s)[F'(\eta)/\eta] \tag{5}$$

Equation (1) is satisfied by introducing a stream function, such that $ur = \psi_r$ and $vr = -\psi_x$, and therefore, with Eqs. (4a) and (5), it follows that $\psi = u_e N^2 F$.

By utilizing well-known operations, Eq. (2) reduces to

$$\left[\eta\left(\frac{F'}{\eta}\right)'\right]' + \frac{1}{\nu}\frac{d(N^2u_e)}{ds} F\left(\frac{F'}{\eta}\right)' + \frac{N^2}{\nu}\frac{du_e}{ds}\eta\left[1 - \left(\frac{F'}{\eta}\right)^2\right] = 0 \tag{6}$$

Equation (6) is subject to the boundary conditions

$$\text{at } \eta = 0 \qquad F = 0 \qquad (F'/\eta)' = 0 \tag{7a}$$

and

$$\text{as } \eta = \infty \qquad (F'/\eta) = 1 \tag{7b}$$

The requirements for similarity are

$$(1/\nu)[d(N^2u_e)/ds] = \alpha = \text{const} \tag{8a}$$

$$(N^2/\nu)(du_e/ds) = \beta = \text{const} \tag{8b}$$

and

$$\text{at } \eta = 0 \qquad F'/\eta = \text{const} \tag{9}$$

Two variations of $u_e(x)$ can be derived from conditions (8). If α is nonzero, then, without loss in generality, it can be set equal to unity. With $\alpha = 1$, Eqs. (8a) and (8b), with (4b), yield, respectively,

$$N^2u_e = \nu(x - x_c) \qquad u_e = u_{e_c}(x/x_c)^\beta \tag{10}$$

where subscript c denotes conditions at an initial station.

If $\alpha = 0$, then Eq. (8a) gives

$$N^2 = \nu/u_e \tag{11a}$$

whereas (8b), with Eq. (4b), yields

$$u_e = u_{e_c}e^{\beta(x-x_c)} \tag{11b}$$

Condition (9), in all cases, implies that u_0/u_e is a constant. (Here u_0 is the streamwise velocity at $r = 0$.)

In general, in order to obtain solutions of the system (6) and (7), numerical methods are required. However, it can readily be shown that there exists one nontrivial closed form exact solution, namely, for $\alpha = 1.0$ and $\beta = 1.0$, it follows that

$$u_e = u_{e_c}(x/x_c) \tag{12a}$$

$$F = (\eta^2/2) - 6(1 - e^{-\eta^2/4}) \tag{12b}$$

and

$$F'/\eta = 1 - 3e^{-\eta^2/4} \tag{12c}$$

References

[1] Steiger, M. H. and Bloom, M. H., "Linearized viscous free-mixing with streamwise pressure gradient," AIAA J. 2, 263–266 (1964).

[2] Napolitano, L., "Influence of pressure gradients on non-homogeneous dissipative free flows," AIAA Preprint 64-100 (January 1964).

Interaction of an Oscillating Magnetic Field with Fluid in Couette Flow

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THE problem of Couette flow interaction with an oscillating transverse magnetic field is solved by means of a transformation that demonstrates the similarity between a set of transient MHD systems and a set of transient heat-conduction systems. Consider two concentric nonconducting cylinders with the inner cylinder at rest. The electrodes, situated at both ends of the annulus, are short-circuited by means of an external connection. For convenience, the distance between the cylinders is assumed small compared to either of the radii. Curvature effects are thus neglected, and the problem is solved in the Cartesian coordinate system. Thus, nonconducting stationary and moving plates are located at $y = 0$ and $y = h$, respectively. A sinusoidal magnetic field B_y is applied perpendicular to the plate. An electrically conducting fluid is in laminar flow, with velocity $u(y,t)$, between the plates. All physical quantities will be considered, at most, functions of y and t.

The interaction of the velocity field $\mathbf{u}$ with the magnetic field $\mathbf{B}$ produces an electric field $\mathbf{E}$, which is given in direction and magnitude by the vector relation $\mathbf{E} = \mathbf{u} \times \mathbf{B}$. The induced current is given by Ohm's law $\mathbf{J} = \sigma\mathbf{E}$, where σ is the electrical conductivity. The magnitude and direction of the Lorentz body force in the momentum equation is given by $\mathbf{J} \times \mathbf{B}$. The magnitude and direction of the electric field vary with the same frequency of the magnetic field. However, as indicated by the double cross-product, the Lorentz force opposes the fluid motion throughout the total cycle of the magnetic field; the frequency associated with the electric field is twice that of the magnetic field. For convenience, the magnetic Reynolds number is taken to be much less than unity. It is interesting to note that this force system, composed of the moving plate and the magnetic field, is quite unsymmetric as far as the fluid is concerned. That is, every particle of moving fluid senses the magnetic force instantaneously (a sort of democratic force), whereas, in the instant the magnetic field is off, the particles of fluid sense the force associated with the moving plate by means of their neighbors, which are between them and the moving plate (an aristocratic force). Strictly speaking, the first mechanism propagates with the velocity of light, whereas the second mechanism propagates with the rate of diffusion of vorticity, which depends on viscosity.

The equation of momentum in the direction of flow is

$$\partial u/\partial t = \alpha(\partial^2 u/\partial y^2) - \alpha N_H^2\, u \sin^2\omega t \tag{1}$$

where

u = velocity
t = time
α $\equiv$ $\mu/\rho h^2$
μ = fluid viscosity
ρ = fluid density
h = distance between plates
y = dimensionless distance from the bottom plate
N_H = Hartmann number
ω = frequency associated with the magnetic field

The boundary and initial conditions are

$$u(0, t) = 0 \qquad u(1, t) = 1 \qquad u(y, 0) = y \tag{2}$$

The limiting form of the solution to Eqs. (1) and (2) when $\omega \to 0$ is obtained by using the transformation $\tau = \omega t$ and neglecting the acceleration term

$$\underset{\omega\to 0}{\mathcal{L}}\; u(y, t) = \frac{\sinh(N_H \sin\omega\, t)y}{\sinh(N_H \sin\omega\, t)} \tag{3}$$

The velocity profile then oscillates between two steady states with a frequency 2ω, responding instantaneously to the changing magnetic field.

The general solution to Eqs. (1) and (2) may be obtained by means of the transformation

$$u(y, t) = V(y, t) \exp - \beta t + \frac{\beta \sin 2\,\omega t}{2\omega} \tag{4}$$

where $\beta \equiv \alpha\, N_H^2/2$. Equations (1) and (2) then reduce to

$$\frac{\partial V}{\partial t} = \alpha \frac{\partial^2 V}{\partial y^2} \tag{5}$$

$$\left.\begin{aligned} V(0, t) &= 0 \\ V(l, t) &= \exp \beta t - \frac{\beta \sin 2\,\omega t}{2\omega} \\ V(y, 0) &= y \end{aligned}\right\} \tag{6}$$

The foregoing system also describes heat conduction between two infinite parallel plates between which a linear temperature exists initially, and, at $t = 0$, the temperature on one boundary is made to vary exponentially in time. The solution to Eqs. (4–6) is[1]

$$u(y, t) = 2 \sum_{n=1}^{\infty} (-1)^{n-1} \sin n\pi\, y \times \exp\left[-\alpha\left(n^2\pi^2 + \frac{N_H^2}{2}\right)t + \frac{\beta}{2}\sin 2\,\omega t\right]\left\{\frac{1}{n\pi} + \alpha\, n\pi \times \int_0^t \exp\left[-\alpha\left(n^2\pi^2 + \frac{N_H^2}{2}\right)t' - \frac{\beta}{2}\sin 2\,\omega t'\right]dt'\right\} \tag{7}$$

When ω is large so that $\exp\,(\beta/2\omega)\,\sin 2\,\omega t$ is negligible, Eq. (7) can be easily integrated:

$$u(y, t) = \frac{\sinh(2^{1/2}/2)\, N_H y}{\sinh(2^{1/2}/2)\, N_H} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin n\,\pi y\, (N_H^2/2) \exp -\alpha[n^2\pi^2 + (N_H^2/2)]t}{(n\pi)[n^2\pi^2 + (N_H^2/2)]} \tag{8}$$

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